Cuaderno de notas de trabajo

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Cuaderno 14

Elemento de Volumen (tridimensional) en el Cono de Luz:

$$(t-\tau)^2-(x-\xi)^2-(y-\eta)^2-(z-\zeta)^2=0.$$

Vértice del cono: τ, ξ, η, ζ.

Coordenadas curvilíneas en el cono:

r, θ, φ;

$$r=\sqrt{(x-\xi)^2+(y-\eta)^2+(z-\zeta)^2}.$$

$$t=\tau+r;$$

$$x=\xi+r\,\mathrm{sen}\,\theta\cos\varphi;$$

$$y=\eta+r\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi;$$

$$z=\zeta+r\cos\theta.$$

$$\varepsilon_{ijkl}=\sqrt{-g}\,e_{ijkl}=e_{ijkl}.$$

$$dV_i=\varepsilon_{ijkl}\,dx^j\,\delta x^k\,\partial x^l=e_{ijkl}\,dx^j\,\delta x^k\,\partial x^l.$$

dx^j corresponde a $d\theta=0$ y $d\varphi=0$; $dr\neq 0$

δx^k corresponde a $dr=0$ y $d\varphi=0$; $d\theta\neq 0$

∂x^l corresponde a $dr=0$ y $d\theta=0$; $d\varphi\neq 0$

$$dx^j = \{ dr\ ,\ \operatorname{sen}\theta\cos\varphi\, dr,\ \operatorname{sen}\theta\operatorname{sen}\varphi\, dr\ ,\ \cos\theta\, dr \}$$

$$\delta x^k = \{ 0,\ r\cos\theta\cos\varphi\, d\theta,\ r\cos\theta\operatorname{sen}\varphi\, d\theta,\ -r\operatorname{sen}\theta\, d\theta \}$$

$$\partial x^l = \{ 0,\ -r\operatorname{sen}\theta\operatorname{sen}\varphi\, d\varphi,\ +r\operatorname{sen}\theta\cos\varphi\, d\varphi,\ 0 \}.$$

$$dV_1 = \cancel{i}\, r^2 dr\, d\theta\, d\varphi \begin{vmatrix} \operatorname{sen}\theta\cos\varphi & \operatorname{sen}\theta\operatorname{sen}\varphi & \cos\theta \\ \cos\theta\cos\varphi & \cos\theta\operatorname{sen}\varphi & -\operatorname{sen}\theta \\ -\operatorname{sen}\theta\operatorname{sen}\varphi & \operatorname{sen}\theta\cos\varphi & 0 \end{vmatrix}$$

$$dV_1 = \cancel{i}\, r^2 dr\, d\theta\, d\varphi \left[\begin{aligned} &\cos\theta \langle \operatorname{sen}\theta\cos\theta\cos^2\varphi + \operatorname{sen}\theta\cos\theta\operatorname{sen}^2\varphi \rangle \\ &+ \operatorname{sen}\theta \langle \operatorname{sen}^2\theta\cos^2\varphi + \operatorname{sen}^2\theta\operatorname{sen}^2\varphi \rangle \end{aligned} \right]$$

$$dV_1 = \cancel{i}\, r^2 dr\, d\theta\, d\varphi \left[\operatorname{sen}\theta\cos^2\theta + \operatorname{sen}^3\theta \right]$$

$$\boxed{dV_1 = \cancel{i}\, r^2 \operatorname{sen}\theta\, dr\, d\theta\, d\varphi}$$

$$dV_2 = -\cancel{i}\, r^2 dr\, d\theta\, d\varphi \begin{vmatrix} 1 & \operatorname{sen}\theta\operatorname{sen}\varphi & \cos\theta \\ 0 & \cos\theta\operatorname{sen}\varphi & -\operatorname{sen}\theta \\ 0 & \operatorname{sen}\theta\cos\varphi & 0 \end{vmatrix}$$

$$dV_2 = -i\, r^2\, dr\, d\theta\, d\varphi\ \mathrm{sen}^2\theta \cos\varphi$$

$$\boxed{dV_2 = -i\, r^2\, \mathrm{sen}^2\theta \cos\varphi\ dr\, d\theta\, d\varphi}$$

$$dV_3 = +i\, r^2\, dr\, d\theta\, d\varphi \begin{vmatrix} 1 & \mathrm{sen}\theta\cos\varphi & \cos\theta \\ 0 & \cos\theta\cos\varphi & -\mathrm{sen}\,\theta \\ 0 & -\mathrm{sen}\theta\,\mathrm{sen}\varphi & 0 \end{vmatrix}$$

$$\boxed{dV_3 = -i\, r^2\, \mathrm{sen}^2\theta\, \mathrm{sen}\varphi\ dr\, d\theta\, d\varphi}$$

$$dV_4 = -i\, r^2\, dr\, d\theta\, d\varphi \begin{vmatrix} 1 & \mathrm{sen}\theta\cos\varphi & \mathrm{sen}\theta\,\mathrm{sen}\varphi \\ 0 & \cos\theta\cos\varphi & \cos\theta\,\mathrm{sen}\varphi \\ 0 & -\mathrm{sen}\theta\,\mathrm{sen}\varphi & \mathrm{sen}\theta\cos\varphi \end{vmatrix}$$

$$dV_4 = -i\, r^2\, dr\, d\theta\, d\varphi\, (\mathrm{sen}\theta\cos\theta\cos^2\varphi + \mathrm{sen}\theta\cos\theta\,\mathrm{sen}^2\varphi)$$

$$\boxed{dV_4 = -i\, r^2\, \mathrm{sen}\theta\cos\theta\ dr\, d\theta\, d\varphi}$$

Elemento de volumen en el cono de luz:

$$t = \tau + r \,;$$
$$x = \xi + r \,\mathrm{sen}\,\theta \cos\varphi \,;$$
$$y = \eta + r \,\mathrm{sen}\,\theta \,\mathrm{sen}\,\varphi \,;$$
$$z = \zeta + r \cos\theta \,.$$

es:

$$dV_i = i\, r^2 dr\, d\theta\, d\varphi \left\{ \mathrm{sen}\,\theta,\ -\mathrm{sen}^2\theta \cos\varphi,\ -\mathrm{sen}^2\theta\,\mathrm{sen}\,\varphi,\ \mathrm{sen}\,\theta\cos\theta \right\}$$

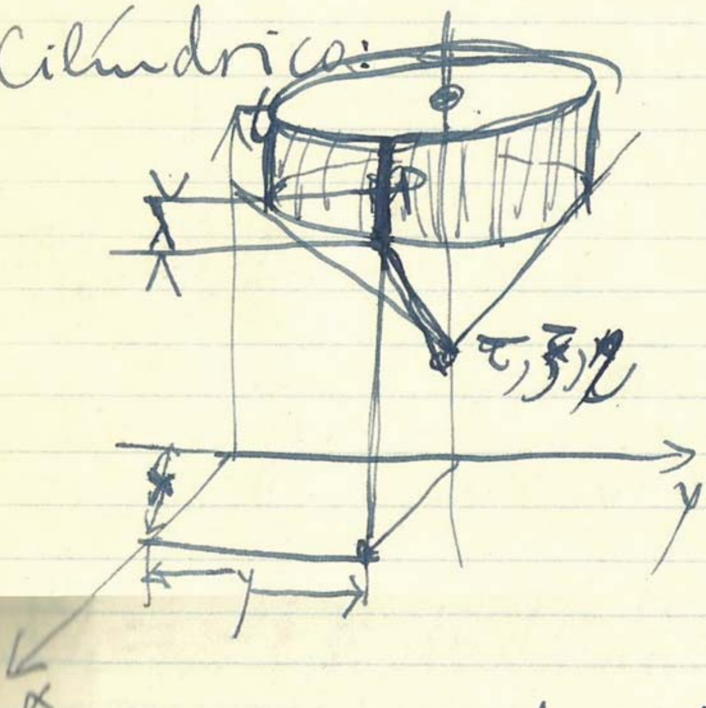
$$\boxed{dV_i = i\, r^2 dr\, d\theta\, d\varphi \left\{ \mathrm{sen}\,\theta,\ -\mathrm{sen}^2\theta \cos\varphi,\ -\mathrm{sen}^2\theta\,\mathrm{sen}\,\varphi,\ -\mathrm{sen}\,\theta\cos\theta \right\}}$$

Comprobado

En la forma en que está escrito dV_i la componente a lo largo del eje del tiempo es un ~~número~~ positivo ~~por i~~ ya que $0 \leqslant \theta \leqslant \pi$.

Elemento de Volumen del Manto

Cilíndrico: Coordenadas: λ, θ, φ



$$t = \tau + A + \lambda \quad \text{parámetro variable} \qquad 0 \leq \lambda \leq \varepsilon$$

$$x = \xi + A \operatorname{sen}\theta \cos\varphi$$

$$y = \eta + A \operatorname{sen}\theta \operatorname{sen}\varphi$$

$$z = \zeta + A \cos\theta$$

$$dx^j = (\ d\lambda\ ,\ 0,\ 0,\ 0)$$

$$\delta x^k = (\ 0,\ A\cos\theta\cos\varphi\, d\theta,\ A\cos\theta \operatorname{sen}\varphi\, d\theta,\ -A \operatorname{sen}\theta\, d\theta)$$

$$\vartheta x^l = (\ 0,\ -A \operatorname{sen}\theta \operatorname{sen}\varphi\, d\varphi,\ A \operatorname{sen}\theta\cos\varphi\, d\varphi,\ 0\ )$$

$$\boxed{dV_1 = 0}$$

menos

$$dV_2 = -\begin{vmatrix} d\lambda & 0 & 0 \\ 0 & \cos\theta \, \mathrm{sen}\,\varphi & -\mathrm{sen}\,\theta \\ 0 & \mathrm{sen}\,\theta \cos\varphi & 0 \end{vmatrix} (-i) A^2 \, d\theta \, d\varphi$$

$$dV_2 = - i A^2 \, d\theta \, d\varphi \, d\lambda \; \mathrm{sen}^2\theta \cos\varphi$$

$$\boxed{dV_2 = - i A^2 \, \mathrm{sen}^2\theta \cos\varphi \, d\lambda \, d\theta \, d\varphi}$$

$$dV_3 = \begin{vmatrix} d\lambda & 0 & 0 \\ 0 & \cos\theta \cos\varphi & -\mathrm{sen}\,\theta \\ 0 & -\mathrm{sen}\,\theta \, \mathrm{sen}\,\varphi & 0 \end{vmatrix} i A^2 \, d\theta \, d\varphi$$

$$dV_3 = - i A^2 \, \mathrm{sen}^2\theta \, \mathrm{sen}\,\varphi \, d\lambda \, d\theta \, d\varphi$$

$$\boxed{dV_3 = - i A^2 \, \mathrm{sen}^2\theta \, \mathrm{sen}\,\varphi \, d\lambda \, d\theta \, d\varphi}$$

$$dV_4 = -i \begin{vmatrix} d\lambda & 0 & 0 \\ 0 & A\cos\theta\cos\varphi\, d\theta & A\cos\theta\,\mathrm{sen}\,\varphi\, d\theta \\ 0 & -A\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi\, d\varphi & A\,\mathrm{sen}\,\theta\cos\varphi\, d\varphi \end{vmatrix}$$

$$dV_4 = -i A^2 d\theta\, d\lambda\, d\varphi \left[\mathrm{sen}\,\theta\cos\theta\cos^2\varphi + \mathrm{sen}\,\theta\cos\theta\,\mathrm{sen}^2\varphi \right]$$

$$dV_4 = -i A^2\, \mathrm{sen}\,\theta\cos\theta\, d\lambda\, d\theta\, d\varphi .$$

$$dV_i = i A^2 d\lambda\, d\theta\, d\varphi \left[0, -\mathrm{sen}^2\theta\cos\varphi, -\mathrm{sen}^2\theta\,\mathrm{sen}\,\varphi, -\mathrm{sen}\,\theta\cos\varphi \right]$$

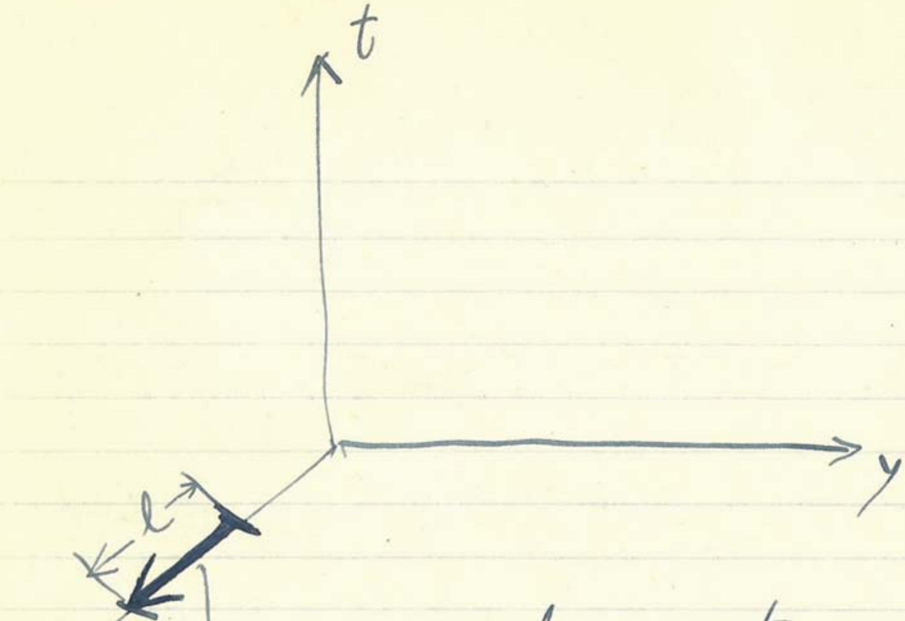
$$\boxed{dV_i = i A^2 d\lambda\, d\theta\, d\varphi \left[0, -\mathrm{sen}^2\theta\cos\varphi, -\mathrm{sen}^2\theta\,\mathrm{sen}\,\varphi, -\mathrm{sen}\,\theta\cos\theta \right]} .$$

Comprobado

Sea e^i un vector unitario a lo largo del eje $O\underline{X}$,

$$e^i = (0, 1, 0, 0)$$

$$e_i = (0, -1, 0, 0)$$



componentes contravariantes $(0, l, 0, 0)$

componentes covariantes $(0, -l, 0, 0)$

$$(0,\ l,\ 0,\ 0)$$

$$(0,\ -\operatorname{sen}^2\theta\cos\varphi,\ -\operatorname{sen}^2\theta\operatorname{sen}\varphi,\ -\operatorname{sen}\theta\cos\theta)$$

$$-l\operatorname{sen}^2\theta\cos\varphi$$

$$dV^i = (0,\ \operatorname{sen}^2\theta\cos\varphi,\ \operatorname{sen}^2\theta\operatorname{sen}\varphi,\ +\operatorname{sen}\theta\cos\theta)$$

Normal hacia afuera.

Cálculo del cuadrigradiente de Ω

$$\boxed{\sigma\Psi(\tau)} = \Omega$$

$$\hat{\nabla}\sigma\Psi(\tau) = (\hat{\nabla}\sigma)\Psi(\tau) + \sigma\Psi'(\tau)\hat{\nabla}\tau$$

$$\boxed{\hat{\nabla}\Omega = \Psi(\tau)\hat{\nabla}\sigma + \sigma\Psi'(\tau)\hat{\nabla}\tau}$$

$$\sigma = \frac{1}{r - \vec{r}\cdot\vec{v}}$$

$$\frac{\partial\sigma}{\partial t} = \sigma^3\left[r(\vec{r}\cdot\vec{a}) + (\vec{r}\cdot\vec{v}) - rv^2\right]$$

$$\nabla\sigma = \sigma^2\vec{v} + \left[-1 + v^2 - (\vec{r}\cdot\vec{a})\right]\sigma^3\vec{r}$$

$$\boxed{\frac{\partial\tau}{\partial t} = r\sigma}$$

$$\nabla\tau = -\vec{r}\sigma$$

Para el caso en que $\vec{v} = 0$

$$\sigma = \frac{1}{r}$$

En el cono de luz:

$$\boxed{\Diamond\sigma = \frac{1}{r^2}\left[(\vec{r}\cdot\vec{a}),\ -(1+\vec{r}\cdot\vec{a})\,\mathrm{sen}\,\theta\cos\varphi,\ -(1+\vec{r}\cdot\vec{a})\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi,\ -(1+\vec{r}\cdot\vec{a})\cos\theta\right]}$$

$$\boxed{\Diamond\tau = \left[1,\ -\mathrm{sen}\,\theta\cos\varphi,\ -\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi,\ -\cos\theta\right]}$$

$$\boxed{\sigma\Diamond\tau = \frac{1}{r}\left[1,\ -\mathrm{sen}\,\theta\cos\varphi,\ -\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi,\ -\cos\theta\right]}$$

$$\frac{\partial\sigma}{\partial t} = \frac{\vec{r}\cdot\vec{a}}{r^2}$$

$$\nabla\sigma = \frac{-1-(\vec{r}\cdot\vec{a})}{r^3}\vec{r}$$

$$\frac{\partial\sigma}{\partial t} = \frac{\vec{r}\cdot\vec{a}}{r^2} \qquad \nabla\sigma = -\frac{1+\vec{r}\cdot\vec{a}}{r^3}\vec{r}.$$

En el cono de luz

$$\hat{\nabla}\sigma = \left(\frac{\vec{r}\cdot\vec{a}}{r^2}, -\frac{1+\vec{r}\cdot\vec{a}}{r^3}r\,\mathrm{sen}\,\theta\cos\varphi, -\frac{1+\vec{r}\cdot\vec{a}}{r^3}r\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi\right.$$
$$\left. -\frac{1+\vec{r}\cdot\vec{a}}{r^3}r\cos\theta\right)$$

$$\hat{\nabla}\sigma = \frac{1}{r^2}\left[\vec{r}\cdot\vec{a}, -(1+\vec{r}\cdot\vec{a})\,\mathrm{sen}\,\theta\cos\varphi, -(1+\vec{r}\cdot\vec{a})\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi\right.$$
$$\left. -(1+\vec{r}\cdot\vec{a})\cos\theta\right]$$

Para el caso en que $\vec{v}=0$

$$\frac{\partial\tau}{\partial t} = 1 \qquad \nabla\tau = -\frac{\vec{r}}{r}$$

En el cono de luz.

$$\hat{\nabla}\tau = (1, -\mathrm{sen}\,\theta\cos\varphi, -\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi, -\cos\theta]$$

En el Manto Cilíndrico

$$\hat{\nabla}\sigma = \left[\frac{\vec{r}\cdot\vec{a}}{A^2},\ -\frac{1+\vec{r}\cdot\vec{a}}{A^2}\,\mathrm{sen}\,\theta\cos\varphi,\ -\frac{1+\vec{r}\cdot\vec{a}}{A^2}\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi,\ -\frac{1+\vec{r}\cdot\vec{a}}{A^2}\cos\theta \right].$$

$$\sigma\hat{\nabla}\tau = \left[\frac{1}{A}\right]\left(1,\ -\mathrm{sen}\,\theta\cos\varphi,\ -\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi,\ -\cos\theta\right]$$

En el Manto Cilíndrico.

$$\frac{\partial\sigma}{\partial t} = \frac{\vec{r}\cdot\vec{a}}{A^2} \qquad \nabla\sigma = -\frac{1+\vec{r}\cdot\vec{a}}{A^3}\vec{r}$$

$$\hat{\nabla}\sigma = \left[\frac{\vec{r}\cdot\vec{a}}{A^2}, -\frac{1+\vec{r}\cdot\vec{a}}{A^2}\operatorname{sen}\theta\cos\varphi, -\frac{1+\vec{r}\cdot\vec{a}}{A^2}\operatorname{sen}\theta\operatorname{sen}\varphi, -\frac{1+\vec{r}\cdot\vec{a}}{A^2}\cos\theta\right]$$

Fl

~~Flujo~~ Elemento de flujo del cuadrigradiente de σ a través del cono de luz.

$$\hat{\nabla}\sigma = \frac{1}{r^2}\left[\vec{r}\cdot\vec{a},\ -(1+\vec{r}\cdot\vec{a})\,\mathrm{sen}\,\theta\cos\varphi,\ -(1+\vec{r}\cdot\vec{a})\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi,\ -(1+\vec{r}\cdot\vec{a})\cos\theta\right]$$

$$dV_i = r^2\,dr\,d\theta\,d\varphi\left[\mathrm{sen}\,\theta,\ -\mathrm{sen}^2\theta\cos\varphi,\ -\mathrm{sen}^2\theta\,\mathrm{sen}\,\varphi,\ -\mathrm{sen}\,\theta\cos\theta\right]$$

Elemento de flujo $d\Phi$

$$d\Phi = dr\,d\theta\,d\varphi\left[\begin{array}{l}\vec{r}\cdot\vec{a}\left\{\mathrm{sen}\,\theta - \overbrace{\overbrace{\mathrm{sen}^3\theta\cos^2\varphi - \mathrm{sen}^3\theta\,\mathrm{sen}^2\varphi}^{-\mathrm{sen}^3\theta} - \mathrm{sen}\,\theta\cos^2\theta}^{-\mathrm{sen}\,\theta}\right\} \\ \left\{\underbrace{\underbrace{-\mathrm{sen}^3\theta\cos^2\varphi - \mathrm{sen}^3\theta\,\mathrm{sen}^2\varphi}_{-\mathrm{sen}^3\theta} - \mathrm{sen}\,\theta\cos^2\theta}\right\}\end{array}\right]$$

$$d\Phi = dr\,d\theta\,d\varphi\left[-\mathrm{sen}\,\theta\right]$$

$$d\Phi = -\mathrm{sen}\,\theta\,dr\,d\theta\,d\varphi$$

Elemento de flujo del cuadrivector gradiente de σ a través del cono de luz.

$$\boxed{d\Phi = -\operatorname{sen}\theta \, dr\, d\theta\, d\varphi}$$

$$\Phi = -\int_0^{2\pi}\int_0^{\pi}\int_0^{A} \operatorname{sen}\theta \, dr\, d\theta\, d\varphi$$

$$\Phi = \cos\theta \Big|_0^{\pi} \; r \Big|_0^{A} \; \varphi \Big|_0^{2\pi}$$

$$\Phi = -2A \cdot 2\pi = -4A\pi$$

$$\boxed{\Phi = -4A\pi}$$

Flujo del cuadrivector gradiente de σ a través del cono de luz

Elemento de Flujo del σ por el cuadrigradiente de τ a través del cono de Luz.

$$\sigma \hat{\nabla} \tau = \frac{1}{r}\left[1, -\operatorname{sen}\theta\cos\varphi, -\operatorname{sen}\theta\operatorname{sen}\varphi, -\cos\theta\right]$$

$$dV_i = r^2\, dr\, d\theta\, d\varphi \left[\operatorname{sen}\theta, -\operatorname{sen}^2\theta\cos\varphi, -\operatorname{sen}^2\theta\operatorname{sen}\varphi, -\operatorname{sen}\theta\cos\theta\right]$$

$$-\operatorname{sen}^3\theta \qquad -\operatorname{sen}\theta$$

$$d\Phi = r\left[\operatorname{sen}\theta - \operatorname{sen}^3\theta\cos^2\varphi - \operatorname{sen}^3\theta\operatorname{sen}^2\varphi - \operatorname{sen}\theta\cos^2\theta\right]$$

$$\cancel{d\Phi = r} \quad d\Phi = 0$$

El flujo del σ por el cuadrigradiente de τ ~~a través~~ a través del cono de luz es cero.

Elemento del flujo del Cuadrigradiente de σ a través del Manto Cilíndrico.

$$\hat{\nabla}\sigma = \left[\frac{\vec{r}\cdot\vec{a}}{A}, -\frac{1+\vec{r}\cdot\vec{a}}{A^2}\operatorname{sen}\theta\cos\varphi, -\frac{1+\vec{r}\cdot\vec{a}}{A^2}\operatorname{sen}\theta\operatorname{sen}\varphi, -\frac{1+\vec{r}\cdot\vec{a}}{A^2}\cos\theta\right]$$

$$dV_i = A^2\, d\lambda\, d\theta\, d\varphi\left[0, -\operatorname{sen}^2\theta\cos\varphi, -\operatorname{sen}^2\theta\operatorname{sen}\varphi, -\operatorname{sen}\theta\cos\theta\right)$$

$$d\Phi = -d\lambda\, d\theta\, d\varphi\left[(1+\vec{r}\cdot\vec{a})(\operatorname{sen}^2\theta + \operatorname{sen}^2\theta\cos^2\theta)\right]$$

$$d\Phi = -\, d\lambda\, d\theta\, d\varphi\,(1+\vec{r}\cdot\vec{a})\operatorname{sen}\theta$$

$$\cancel{\Phi = -\varepsilon\cdot 2\pi\,(1+\vec{r}\cdot\vec{a}) = -4\pi\varepsilon - 4\pi\varepsilon(\vec{r}\cdot\vec{a})}$$

Flujo de ~~cuart~~ σ por el cuadrigradiente de τ a través del manto cilíndrico.

$$\sigma \overset{1}{\nabla} \tau = \frac{1}{A}\left(1, -\operatorname{sen}\theta\cos\varphi, -\operatorname{sen}\theta\operatorname{sen}\varphi, -\cos\theta\right)$$

$$dV_i = A^2 d\lambda\, d\theta\, d\varphi \left[0, -\operatorname{sen}^2\theta\cos\varphi, -\operatorname{sen}^2\theta\operatorname{sen}\varphi, -\operatorname{sen}\theta\cos\theta\right]$$

$$-\operatorname{sen}^3\theta$$

$$d\Phi = A\, d\lambda\, d\theta\, d\varphi \left(0 - \operatorname{sen}^3\theta\cos^2\varphi - \operatorname{sen}^3\theta\operatorname{sen}^2\varphi - \operatorname{sen}\theta\cos^2\theta\right)$$

$$d\Phi = -A\, d\lambda\, d\theta\, d\varphi\, \operatorname{sen}\theta$$

$$\int_0^{\pi} -\operatorname{sen}\theta\, d\theta = \cos\theta \Big|_0^{\pi} = -2$$

$$\Phi = -2A\varepsilon \cdot 2\pi = -4A\pi\varepsilon$$

Definición del Manto Cilíndrico.

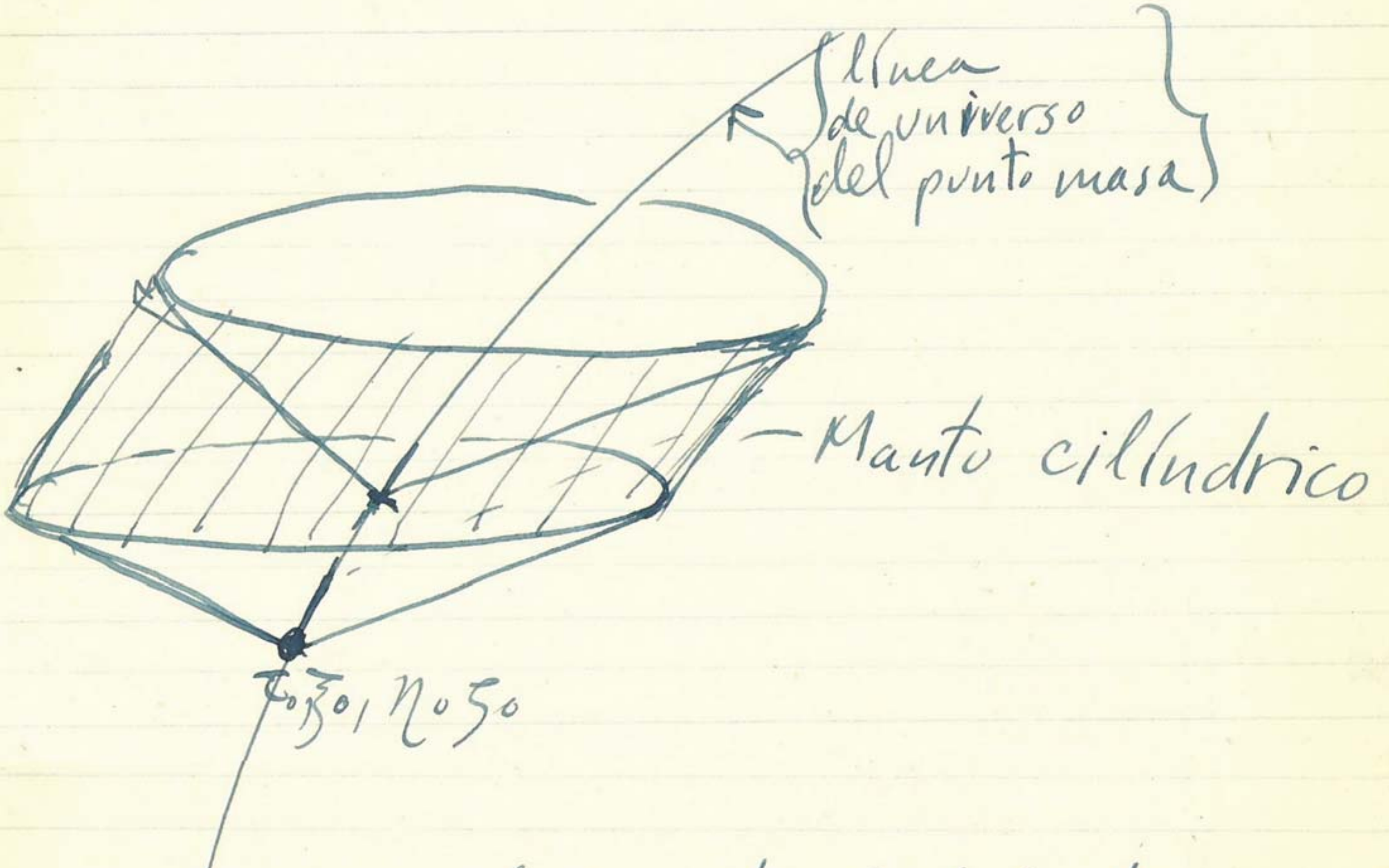
1) Sean τ, ξ, η, ζ las coordenadas de un acontecimiento ~~retardado~~ en la línea de universo del punto masa.

2) Sean $\tau_0, \xi_0, \eta_0, \zeta_0$ un acontecimiento particular en esa línea de universo.

3) Constrúyanse los conos de luz del futuro con vértice en ~~los~~ el punto $\tau_0, \xi_0, \eta_0, \zeta_0$ y los puntos del arco de línea de universo que empieza en ese acontecimiento y acaba en $\tau = \tau_0 + \Delta\tau, \xi, \eta, \zeta$.

4) Córtese el cono de luz con vértice en τ, ξ, η, ζ con el plano $\tau + A$. Este plano corta al cono en una esfera (S_3). El conjunto de las esferas en los planos desde $\tau_0 + A$ hasta $\tau_0 + \Delta\tau + A$

formar el manto cilíndrico.



Ecuaciones del Manto Cilíndrico.

$$t = \tau + A$$
$$x = \xi + A \operatorname{sen}\theta \cos\varphi$$
$$y = \eta + A \operatorname{sen}\theta \operatorname{sen}\varphi$$
$$z = \zeta + A \cos\theta$$

Coordenadas curvilíneas

$$\tau, \theta, \varphi$$

$$\xi = \xi(\tau), \quad \eta = \eta(\tau), \quad \zeta = \zeta(\tau)$$

dx^j ——— sólo varía τ;

δx^k ——— sólo varía θ;

ϑx^l ——— sólo varía φ.

$$dx^j = (1, v^x, v^y, v^z)\, d\tau$$

$$\delta x^k = (0, \cos\theta\cos\varphi, \cos\theta \,\mathrm{sen}\,\varphi, -\mathrm{sen}\,\theta)\, A\, d\theta$$

$$\vartheta x^l = (0, -\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi, \mathrm{sen}\,\theta\cos\varphi, 0)\, A\, d\varphi$$

$$dV_i = \left\| \begin{array}{c|c|c|c} 1 & v^x & v^y & v^z \\ 0 & \cos\theta\cos\varphi & \cos\theta\,\mathrm{sen}\,\varphi & -\mathrm{sen}\,\theta \\ 0 & -\mathrm{sen}\,\varphi & \cos\varphi & 0 \end{array} \right\| A^2 \mathrm{sen}\,\theta\, d\tau\, d\theta\, d\varphi$$

$$\boxed{dV_1 = \left[v^x \mathrm{sen}\,\theta\cos\varphi + v^y \mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi + v^z \cos\theta \right] A^2 \mathrm{sen}\,\theta\, d\tau\, d\theta\, d\varphi}$$

$$dV_2 = - \begin{vmatrix} 1 & v^y & v^z \\ 0 & \cos\theta\,\mathrm{sen}\,\varphi & -\mathrm{sen}\,\theta \\ 0 & \cos\varphi & 0 \end{vmatrix} A^2 \mathrm{sen}\,\theta\, d\tau\, d\theta\, d\varphi$$

$$\boxed{dV_2 = - A^2 \mathrm{sen}^2\theta \cos\varphi\; d\tau\, d\theta\, d\varphi}$$

$$dV_3 = + \begin{vmatrix} 1 & v^x & v^z \\ 0 & \cos\theta\cos\varphi & -\operatorname{sen}\theta \\ 0 & -\operatorname{sen}\varphi & 0 \end{vmatrix} A^2 \operatorname{sen}\theta \, d\tau \, d\theta \, d\varphi$$

$$\boxed{dV_3 = - A^2 \operatorname{sen}^2\theta \operatorname{sen}\varphi \, d\tau \, d\theta \, d\varphi}$$

$$dV_4 = - \begin{vmatrix} 1 & v^x & v^y \\ 0 & \cos\theta\cos\varphi & \cos\theta\cos\varphi \\ 0 & -\operatorname{sen}\varphi & \cos\varphi \end{vmatrix} A^2 \operatorname{sen}\theta \, d\tau \, d\theta \, d\varphi$$

$$\boxed{dV_4 = - A^2 \operatorname{sen}\theta \cos\theta \, d\tau \, d\theta \, d\varphi}$$

Elemento de Volumen del Manto Cilíndrico.

$$dV_1 = \left[v^x \operatorname{sen}\theta\cos\varphi + v^y \operatorname{sen}\theta \operatorname{sen}\varphi + v^z \cos\theta \right] A^2 \operatorname{sen}\theta \, d\tau \, d\theta \, d\varphi$$

$$dV_2 = - A^2 \operatorname{sen}^2\theta \cos\varphi \, d\tau \, d\theta \, d\varphi$$

$$dV_3 = - A^2 \operatorname{sen}^2\theta \operatorname{sen}\varphi \, d\tau \, d\theta \, d\varphi$$

$$dV_4 = - A^2 \operatorname{sen}\theta \cos\theta \, d\tau \, d\theta \, d\varphi$$

Comprobado

$$\frac{\partial}{\partial t}\sigma = \frac{r(\vec{r}\cdot\vec{a})+\vec{r}\cdot\vec{v}-rv^2}{(r-\vec{r}\cdot\vec{v})^3}$$

$$\nabla\sigma = \frac{\vec{v}}{(r-\vec{r}\cdot\vec{v})^2}+\frac{[-1+v^2-\vec{r}\cdot\vec{a}]\,\vec{r}}{(r-\vec{r}\cdot\vec{v})^3}$$

$$dV_1 = (\vec{r}\cdot\vec{v})\,A\,\mathrm{sen}\,\theta\,d\tau\,d\theta\,d\varphi$$

$$dV_2 = -A^2\,\mathrm{sen}^2\theta\cos\varphi\,.\,d\tau\,d\theta\,d\varphi$$

$$dV_3 = -A^2\,\mathrm{sen}^2\theta\,\mathrm{sen}\,\varphi\,.\,d\tau\,d\theta\,d\varphi$$

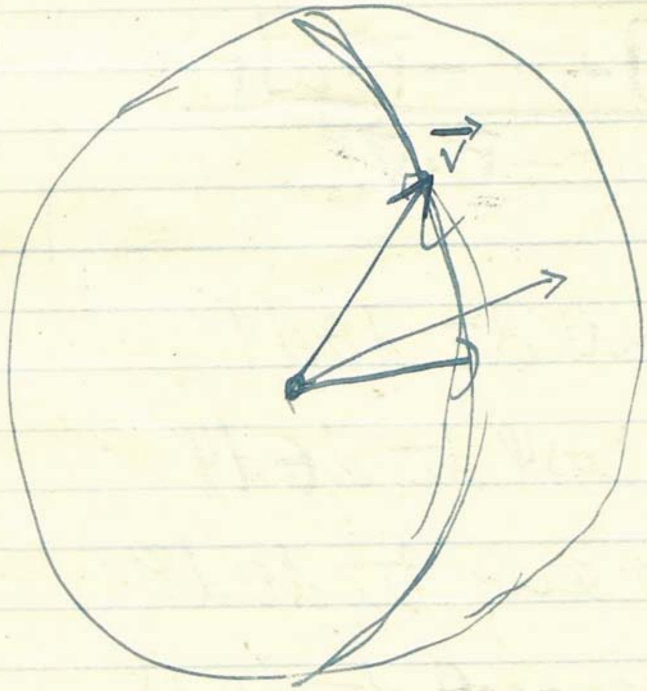
$$dV_4 = -A^2\,\mathrm{sen}\,\theta\cos\theta\,d\tau\,d\theta\,d\varphi$$

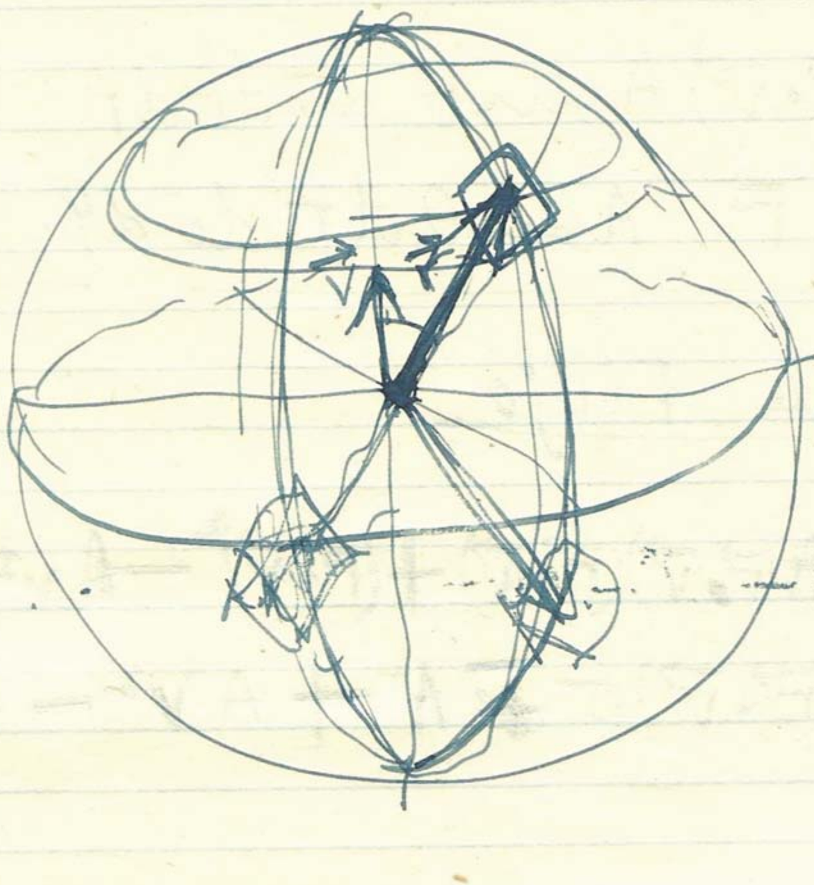
$$dV_1 = (\vec{r}\cdot\vec{v})\,A\,\mathrm{sen}\,\theta\,d\tau\,d\theta\,d\varphi$$

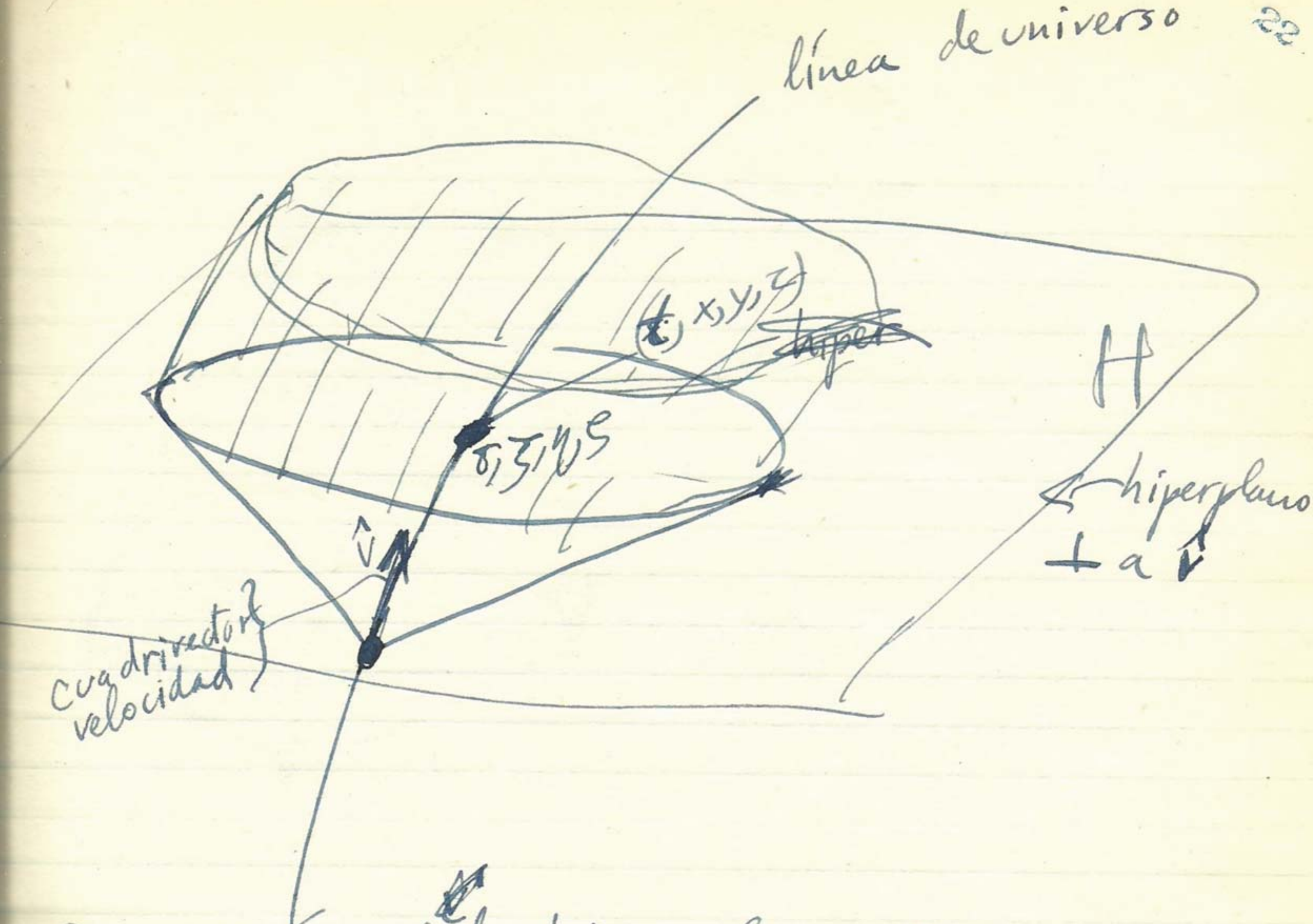
$$d\vec{V} = -\vec{r}\;A\,\mathrm{sen}\,\theta\,d\tau\,d\theta\,d\varphi$$

Elemento de Flujo

$$d\Phi = \frac{A\,\mathrm{sen}\,\theta\,d\tau\,d\theta\,d\varphi}{\sigma^3}\left|\begin{array}{l} A(\vec{r}\cdot\vec{v})(\vec{r}\cdot\vec{a})+(\vec{r}\cdot\vec{v})^2-Av^2(\vec{r}\cdot\vec{v}) \\ (\vec{r}\cdot\vec{v})\,\sigma-A^2+A^2v^2-A^2(\vec{r}\cdot\vec{a}) \end{array}\right.$$


$\vec{v}$





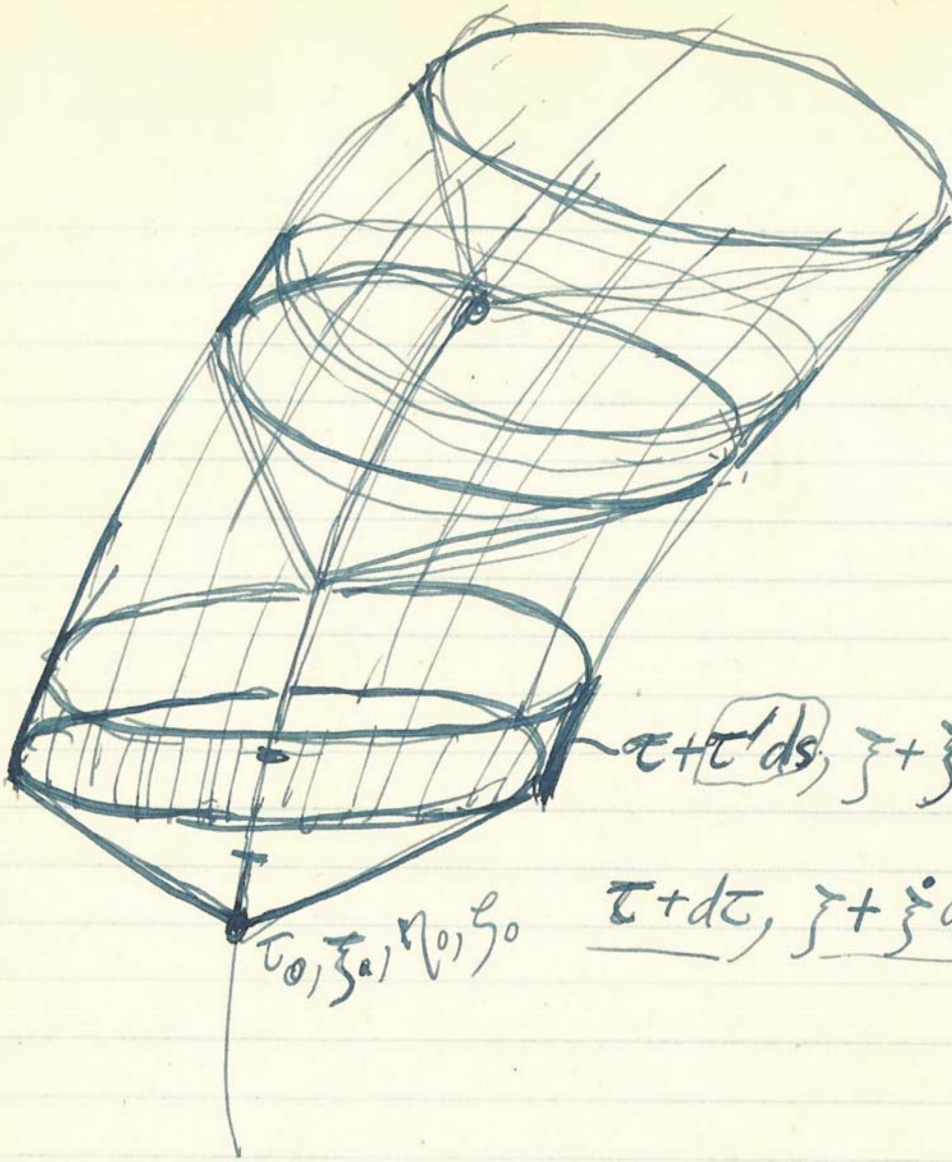
Ecuación del hiperplano H:

$\cancel{(t-\tau)^2}$

$$v^1(t-\tau) - v^2(x-\xi) - v^3(y-\eta) - v^4(z-\varsigma) = 0$$

$$v^1 t - v^2 x - v^3 y - v^4 z = v^1\tau - v^2\xi - v^3\eta - v^4\varsigma$$

$$(t-\tau_0)^2 - (x-\xi_0)^2 - (y-\eta_0)^2 - (z-\varsigma_0)^2$$



$\tau + \boxed{\tau' ds}, \; \xi + \xi' ds, \; \eta + \eta' ds, \; \zeta + \zeta' ds$
$\tau + d\tau, \; \xi + \dot{\xi} d\tau, \; \eta + \dot{\eta} d\tau, \; \zeta + \dot{\zeta} d\tau$
$\tau_0, \xi_0, \eta_0, \zeta_0$

1) Ecuaciones de Transformación de Lorentz.

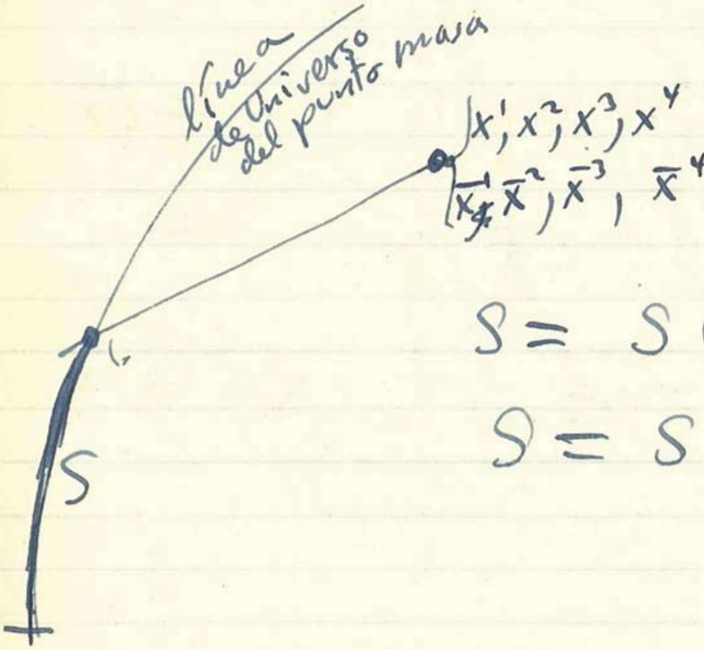
$$\bar{x}^i = a^i_j x^j \qquad x^i = A^i_j \bar{x}^j$$

$$a^i_m A^m_j = \delta^i_j$$

$$a^m_i A^j_m = \delta^i_j$$

$$a A = 1$$

2) Las a^i_j y A^i_j son funciones de S.
S es el arco a lo largo de la línea de universo de una punto-masa.



$$S = S(x^1, x^2, x^3, x^4)$$

$$S = S(\bar{x}^1, \bar{x}^2, \bar{x}^3, \bar{x}^4)$$

$$\bar{x} = \frac{x - vt}{\sqrt{1-v^2}} \qquad \bar{y} = y \qquad \bar{z} = z$$

$$\cancel{\bar{y} = y - vt} \qquad \bar{t} = \frac{t - vx}{\sqrt{1-v^2}}$$

$$\bar{x} = x + \left[-1 + \frac{1}{\sqrt{1-v^2}}\right] x$$

$$\bar{y} = y$$

$$\bar{z} = z$$

$$\overline{\vec{r}} = \vec{r} + \left[\frac{1}{\sqrt{1-v^2}} - 1\right] \frac{\vec{r}\cdot\vec{v}}{v^2}\vec{v}$$

$$\bar{t} = \frac{1}{\sqrt{1-v^2}}\left[t - \vec{r}\cdot\vec{v}\right]$$

$$\bar{t} = \frac{1}{\sqrt{1-v^2}}\left[t - xv^x - yv^y - zv^z\right]$$

$$\bar{x} = x + \frac{1}{v^2}\left[\frac{1}{\sqrt{1-v^2}} - 1\right]\left[xv^x + yv^y + zv^z\right] v^x$$

$$\bar{y} = y + \frac{1}{v^2}\left[\frac{1}{\sqrt{1-v^2}} - 1\right]\left[xv^x + yv^y + zv^z\right] v^y$$

$$\bar{z} = z + \frac{1}{v^2}\left[\frac{1}{\sqrt{1-v^2}} - 1\right]\left[x + v^x + yv^y + zv^z\right] v^z$$

$$\bar{x} = \frac{x - vt}{\sqrt{1-(v)^2}} \qquad \bar{y} = y \qquad \bar{z} = z$$

$$\bar{t} = \frac{t - vx}{\sqrt{1-(v)^2}}$$

$$\bar{x} = x + \left[\frac{1}{\sqrt{1-(v)^2}} - 1\right] x - \frac{v}{\sqrt{1-(v)^2}} t$$

$$\cancel{\bar{x} = -v^2 t + x + \left[\frac{1}{\sqrt{1-(v)^2}} - 1\right] \frac{\vec{r}\cdot\vec{v}}{v}}$$

$$\cancel{\bar{y} = y \qquad \bar{z} = z}$$

$$\cancel{\vec{\bar{r}} = \vec{r} - v^2 t}$$

$$\cancel{\bar{x} = \vec{r}\cdot\vec{v}}$$

$$\vec{\bar{r}} = \vec{r} + \left\langle \left[\frac{1}{\sqrt{1-(v)^2}} - 1\right] \frac{\vec{v}\cdot\vec{r}}{v} - \frac{vt}{\sqrt{1-(v)^2}} \right\rangle \frac{\vec{v}}{v}$$

$$\vec{\bar{r}} = \vec{r} + \left\langle \left\{\frac{1}{\sqrt{1-(v)^2}} - 1\right\} \frac{\vec{v}\cdot\vec{r}}{v^2}\vec{v} - \frac{t\vec{v}}{\sqrt{1-(v)^2}} \right\rangle$$

$$\bar{t} = \frac{t}{\sqrt{1-(v)^2}} - \frac{\vec{r}\cdot\vec{v}}{\sqrt{1-(v)^2}}$$

$$-1$$

$$\cancel{(v^x)^2}\left[\frac{1}{1-(v)^2} - \frac{1}{(v)^2}\left\{\frac{1}{1-(v)^2} - \frac{2}{\sqrt{1-(v)^2}} + 1\right\} - \frac{2}{(v)^2}\left\{\frac{1}{\sqrt{1-(v)^2}} - 1\right\}\right]$$

$$(v^x)^2\left[\frac{1}{1-(v)^2} - \frac{1}{(v)^2(1-(v)^2)} - \frac{1}{(v)^2} + \frac{2}{(v)^2}\right]$$

$$(v^x)^2\left[\frac{v^2-1+(1-v^2)}{(v)^2[1-(v)^2]}\right] = 0$$

$$\frac{v^x v^y}{1-v^2} - \left\{1 + \frac{(v^x)^2}{v^2}\left\langle\frac{1}{\sqrt{1-v^2}} - 1\right\rangle\right\}\left\{\frac{v^x v^y}{v^2}\left\langle\frac{1}{\sqrt{1-v^2}} - 1\right\rangle\right\}$$

$$-\left\{\frac{v^x v^y}{v^2}\left\langle\frac{1}{\sqrt{1-v^2}} - 1\right\rangle\right\}\left\{1 + \frac{(v^y)^2}{v^2}\left\langle\frac{1}{\sqrt{1-v^2}} - 1\right\rangle\right\}$$

$$-\left\{\frac{v^x v^z}{v^2}\left\langle\frac{1}{\sqrt{1-v^2}} - 1\right\rangle\right\}\left\{\frac{v^y v^z}{v^2}\left\langle\frac{1}{\sqrt{1-v^2}} - 1\right\rangle\right\} =$$

$$\frac{v^x v^y}{1-v^2} - \left(\frac{v^x v^y}{v^2}\right)\left\langle\frac{1}{\sqrt{1-v^2}} - 1\right\rangle^2 - 2\frac{v^x v^y}{v^2}\left\langle\frac{1}{\sqrt{1-v^2}} - 1\right\rangle$$

$$\frac{v^x v^y}{v^2(1-v^2)}\left[v^2 - (1-v^2)\left\langle\frac{1}{1-v^2} - \frac{2}{\sqrt{1-v^2}} + 1\right\rangle - 2(1-v^2)\left(\frac{1}{\sqrt{1-v^2}} - 1\right)\right]$$

$$\frac{v^x v^y}{v^2(1-v^2)}\left[\cancel{v^2} - \cancel{1} - 1 + \cancel{v^2} + \cancel{2} - \cancel{2v^2}\right] = 0$$

$$\cancel{\bar{x}\, t = x}$$

$$\bar{x} = -\frac{v^x}{\sqrt{1-(v)^2}}\,t + x + \frac{v^x}{v^2}\left\{\frac{1}{\sqrt{1-(v)^2}} - 1\right\}\left\{x v^x + y v^y + z v^z\right\}$$

$$\bar{y} = -\frac{v^y}{\sqrt{1-(v)^2}}\,t + y + \frac{v^y}{v^2}\left\{\frac{1}{\sqrt{1-(v)^2}} - 1\right\}\left\{x v^x + y v^y + z v^z\right\}$$

$$\bar{z} = -\frac{v^z}{\sqrt{1-(v)^2}}\,t + z + \frac{v^z}{v^2}\left\{\frac{1}{\sqrt{1-(v)^2}} - 1\right\}\left\{x v^x + y v^y + z v^z\right\}$$

$$\cancel{\bar{t} = \left\{\frac{t}{\sqrt{1-v^2}}\right\} - v\cdot x}$$

$$\bar{t} = \frac{t}{\sqrt{1-(v)^2}} - \frac{x v^x + y v^y + z v^z}{\sqrt{1-(v)^2}}$$

$$\cancel{\frac{(v^x)^2}{1-(v)^2} - 1 - \frac{(v^x)^2}{v^4}\left\{\frac{1}{\sqrt{1-(v)^2}} - 1\right\}^2\left\{x v^x + y v^y + z v^z\right\}^2}$$

$$\frac{(v^x)^2}{1-(v)^2} - \left[1 + \frac{(v^x)^2}{(v)^2}\left\{\frac{1}{\sqrt{1-(v)^2}} - 1\right\}\right]^2 - \left[\frac{v^x v^y}{(v)^2}\left\{\frac{1}{\sqrt{1-(v)^2}} - 1\right\}\right]^2$$

$$-\left[\frac{v^x v^z}{(v)^2}\left\{\frac{1}{\sqrt{1-(v)^2}} - 1\right\}\right]^2 =$$

$$\left(\frac{(v^x)^2}{1-(v)^2} - \frac{(v^x)^2\left\langle (v^x)^2 + (v^y)^2 + (v^z)^2\right\rangle}{(v)^4}\left\{\frac{1}{\sqrt{1-(v)^2}} - 1\right\}^2 - 1 - \frac{2(v^x)^2}{(v)^2}\left\{\frac{1}{\sqrt{1-(v)^2}} - 1\right\}\right) = -1$$

$$\bar{t} = \frac{t}{\sqrt{1-v^2}}\, t - \frac{xv^x + yv^y + zv^z}{\sqrt{1-v^2}}$$

$$\bar{x} = -\frac{v^x}{\sqrt{1-v^2}}\, t + x + \frac{v^x}{v^2}\left\{\frac{1}{\sqrt{1-v^2}} - 1\right\}\left\{xv^x + yv^y + zv^z\right\}$$

$$\bar{y} = -\frac{v^y}{\sqrt{1-v^2}}\, t + y + \frac{v^y}{v^2}\left\{\frac{1}{\sqrt{1-v^2}} - 1\right\}\left\{xv^x + yv^y + zv^z\right\}$$

$$\bar{z} = -\frac{v^z}{\sqrt{1-v^2}}\, t + z + \frac{v^z}{v^2}\left\{\frac{1}{\sqrt{1-v^2}} - 1\right\}\left\{xv^x + yv^y + zv^z\right\}$$

$$\cancel{\frac{v^x v^x}{1-v^2}} \qquad 1 + \frac{v^x v^x}{v^2}\left\{\frac{1}{\sqrt{1-v^2}} - 1\right\}$$

$$1\cancel{A}\ \cancel{\frac{v^2 v^2 (1-v^2)}{v^2}}\left\{\frac{1}{\sqrt{1-v^2}} - 1\right\}$$